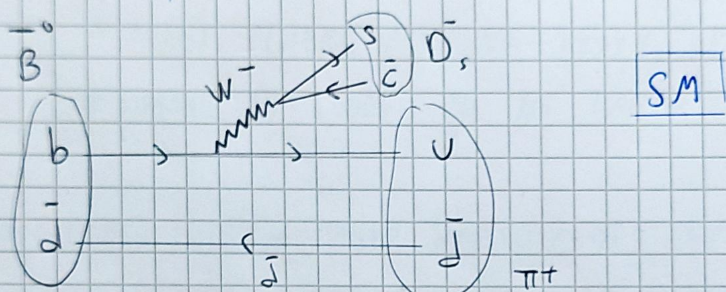
OPERATOR MIXING IN LEFT

THE LEFT IS PARTICULARLY USEFUL TO COMPUTE LOW-ENERGY PROCESS SUCH AS B-MESON DECAYS

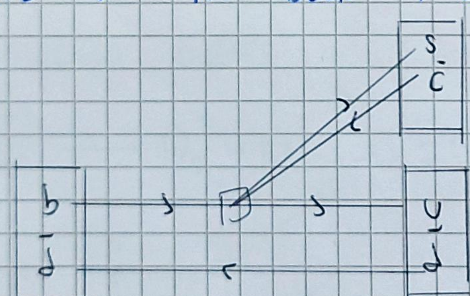
$$\bar{B}^0 \rightarrow D_s^- + \pi^+$$

$$\bar{B}^0 = (\bar{d}\, b) \qquad D_s^- = (s\,\bar{c}) \qquad \pi^+ = (u\,\bar{d})$$

IN THE SM, THE RELEVANT FEYNMAN DIAGRAM IS



SINCE $m_{\bar{B}^0} \ll m_W$, WE CAN DESCRIBE THE PROCESS IN TERMS OF 4 FERMION OPERATORS WHERE W^- HAS BEEN INTEGRATED OUT



SO AMPLITUDE FOR THIS DECAY IS PROPORTIONAL TO $\propto m_b^2 / M_W^2$

ONE CAN ARGUE THAT HIGHER ORDER TERMS IN THE EFT EXPANSION ARE SUPPRESSED & CAN BE NEGLECTED

HOWEVER IF WE COMPUTE LOOP CORRECTIONS TO THIS PROCESS IN THE SM WE FIND

CORRECTIONS OF FORM

$$\alpha_s \ln\left(\frac{M_W^2}{m_b^2}\right) \sim 0.2 \times \ln(250) \sim 0$$

Q1 WHAT DOES THIS RESULT TELL US

→ EFFECTIVELY "TOO LARGE" EXPANSION PARAMET

Q2 WOULD THIS KIND OF EFFECTS ARISE IN LEFT CALCULATIONS?

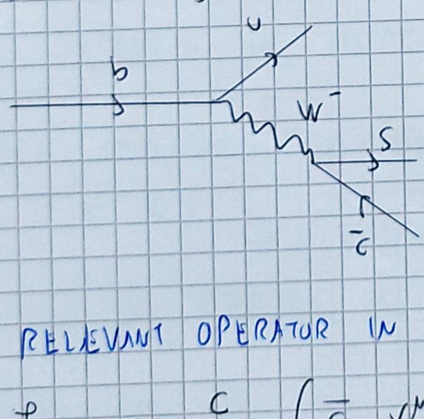
↳ NO, THERE ARE NO HEAVY PARTICLES FLOWING IN LOOPS SO NO LARGE LOGARIT

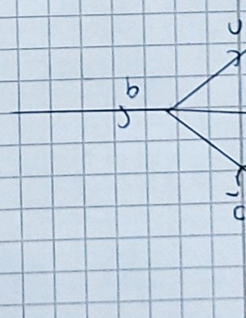
→ RESUM LARGE LOGS + KEEP SIMPLICITY OF LEFT DESC BY USING "RG-IMPROVED PERTURBATION THEORY"

↳ ONLY EXPANSION PARAMETER IS $m_b^2/M_W^2 \sim 3 \times 1$

LET'S APPLY THIS IDEAS TO THE B-MESON DECAY CONSIDERED BEFORE

$\bar{B}^0 \longrightarrow D_s^- + \pi^+$ SM LEFT





THE RELEVANT OPERATOR IN LEFT IS

$$\mathcal{L}_{LEFT} \supset \frac{c}{\Lambda^2}\left(\bar{s}_L \gamma^\mu c_L\right)\left(\bar{u}_L \gamma_\mu b_L\right)$$

$(\bar{L}L)(\bar{L}L)$ - TYPE

WE CAN EVALUATE THE WILSON COEFFICIENT BY MATCHING TO SM

$$\frac{C}{\Lambda^2} = -\frac{4 G_F}{\sqrt{2}} V_{cs}^* V_{ub}$$

[IN THE PRESENCE OF BSM MATCHING WOULD BE DIFFERENT]

ONCE WE ACCOUNT FOR COLOR INDICES (SEE NOTES) WE FIND THAT WE ACTUALLY HAVE TWO OPERATORS RELEVANT TO DESCRIBE THIS DECAY

$$\mathcal{L}_{EFT} \supset L_1 O_1 + L_2 O_2$$

$$L_1 = -\frac{1}{N_c} \frac{4 G_F}{\sqrt{2}} V_{cs}^* V_{ub}$$ [TREE-LEVEL MATCHING WITH SM]

$$O_1 = (\bar{s}_L \gamma^\mu b_L)(\bar{u}_L \gamma_\mu c_L)$$

$$L_2 = -\frac{8 G_F}{\sqrt{2}} V_{cs}^* V_{ub}$$ [TREE-LEVEL MATCHING WITH SM]

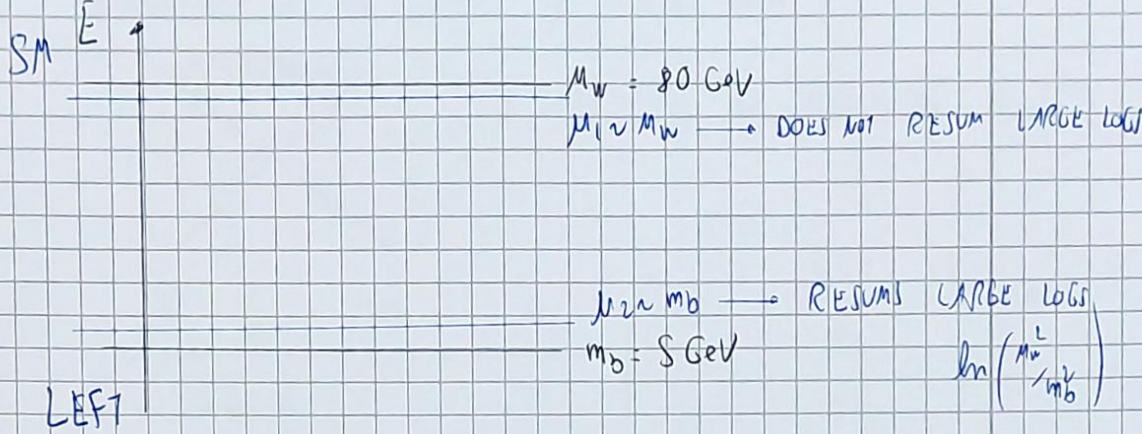
$$O_2 = (\bar{s}_L \gamma^\mu T^A b_L)(\bar{u}_L \gamma_\mu T^A c_L)$$

[Q] "HAVE YOU HEARD ABOUT LARGE-N_c EXPANSION IN QCD? IS IT SENSIBLE

SINCE AMPLITUDES DO NOT INTERFERE (DIFFERENT COLOR FLOW) $\Rightarrow$ CONTRIBUTION FROM O_1 IS SUPPRESSED AS $1/N_c^2$ IN DECAY RATES

[Q] IN THIS WAY, WILL WE REPRODUCE IN THE LEFT THE LARGE LOGARITHS $\alpha_s \ln(m_W^2/m_b^2)$ THAT ARISE IN SM $\rightarrow$ NO!

REASON $\longrightarrow$ MATCHING DONE AT $\mu = M_W$



WE CAN USE FLEXIBILITY OFFERED BY LEFT TO MATCH WITH SM AT ANY VALUE WITHIN m_b & m_W

LET'S VERIFY THAT IF WE MATCH AT $\mu_2 \sim m_b$ WE NOT ONLY REPRODUCE, BUT ALSO <u>RESUM TO ALL-ORDERS</u> THE LOGS FROM THE FIXED ORDER "SM" CALCULATION

DEPENDENCE OF WILSON COEFFICIENTS L_i ON MATCHING SCALE μ IS ABSENT AT TREE-LEVEL, ARISES FROM <u>1-LOOP QCD CORRECTIONS</u>

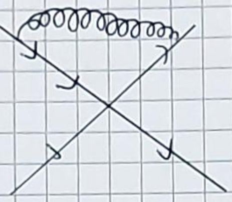
$$\left\{\begin{aligned} \tilde{\psi}_q &= Z_q^{1/2}\,\psi_q \\ \tilde{L}_i &= Z_{ij}\,L_j \end{aligned}\right\} \longrightarrow \text{RENORMALIZED QUANTITIES} \qquad i = 1,2$$

$\longrightarrow$ BARE QUANTITIES

$\longrightarrow$ RENORMALIZATION OF WILSON COEFFICIENTS IS <u>NOT DIAGONAL</u>,

$\longrightarrow$ <u>OPERATOR MIXING</u>

THE CALCULATION OF Z_{ij} REQUIRES ONE-LOOP DIAGRAMS IN THE LEFT OF THE FORM



[HERE STILL NO LARGE-LOGS $\ln\left(\mu_W^2/m_b^2\right)$]

$$\frac{d}{d\ln \underset{\uparrow}{\tilde{\mu}}}\vec{L} = \frac{g_s}{4\pi}\begin{pmatrix} 0 & \frac{6C_F}{N_c} \\ 12 & -12/N_c \end{pmatrix}\vec{L}$$

RENORMALIZATION SCALE

WHAT ARE CONSEQUENCES OF ONE-LOOP QCD CORRECTIONS

ASSUME μ_0 $L_1 = 0,\ L_2 = A$

$$\frac{d}{d\ln\tilde{\mu}}\begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \frac{g_s}{4\pi}\begin{pmatrix} 0 & \frac{6C_F}{N_c} \\ 12 & -12/N_c \end{pmatrix}\begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$$

$$d/d\ln\tilde{\mu}\ L_1 = \frac{6C_F}{N_c} L_2$$

DUE TO <u>OPERATOR MIXING AND RUNNING</u>, EVEN IF $L_1(\mu_0) = 0$, IN GENERAL

$$L_1(\mu_1 \neq \mu_0) \neq 0 \qquad \text{AT ONE-LOOP}$$

WE CAN SOLVE THE RGEs IN THE BASIS THAT MAKES IT DIAGONAL

$$\begin{vmatrix} -\lambda & 8/3 \\ 12 & -4-\lambda \end{vmatrix} = 0$$

$$\lambda(4+\lambda) - 32 = 0$$

$$\lambda^2 + 4\lambda - 32 = 0$$

$$\lambda = \frac{1}{2}\left(-4 \pm \sqrt{16 + 4 \times 32}\right)$$

$$\lambda = \frac{1}{2}\left(-4 \pm \sqrt{144}\right) = \begin{cases} 4 \\ -8 \end{cases}$$

WE COMPUTE THEN CORRESPONDING EIGENVALUES

THE RESULTING RGEs IN THE DIAGONAL BASIS ARE

$$L^{\pm}(\tilde{\mu}) = L^{\pm}(\tilde{\mu}_0 = M_W) \exp\left(\int_{g_s(m_W)}^{g(\tilde{\mu})} dg \, \frac{g}{4\pi} \frac{\gamma^{\pm}}{\beta(g)}\right)$$

$$\gamma^{\pm} = 6\left(\pm 1 - \frac{1}{N_c}\right)$$

QCD BETA FUNCTION

INITIAL CONDITION MATCHED LEFT AND SM AT $\tilde{\mu} \to M_W$

$$L^{\pm}(\tilde{\mu}) = L^{\pm}(m_W)\left(g_s(m_W) / g_s(\tilde{\mu})\right)^{\gamma^{\pm}/2\beta_0}$$

$$L^{\pm}(\tilde{\mu}) = L^{\pm}(m_W)\left(\frac{g_s(m_W)}{g_s(\tilde{\mu})}\right)^{\gamma^{\pm}/2\beta_0}$$

NUMBER

$$\frac{g_s(\tilde{\mu})}{g_s(m_W)} = \frac{1}{1 + g_s(m_W)/4\pi \ \ln\left(\tilde{\mu}^2/m_W^2\right)}$$

$$L^{\pm}(m_b) = L^{\pm}(m_W)\, e^{\gamma^{\pm}/2\beta_0 \underbrace{\ln\left[1 + \frac{g_s(m_W)}{4\pi}\ln\left(\tilde{\mu}^2/m_W^2\right)\right]}_{\simeq\, g_s(m_W)/4\pi \ \ln\left(m_b^2/m_W^2\right)}}$$

RGE RUNNING
DOWN TO $\tilde{\mu} = m_b$

$$L^{\pm}(m_b) = L^{\pm}(m_W)\, e^{\frac{\gamma^{\pm}}{2\beta_0}\frac{g_s(m_W)}{4\pi}\ln\left(m_b^2/m_W^2\right)}$$

LARGE
LOGARITHMS
ACCOUNTED FOR
+
RESUMMED
TO ALL ORDER

$$= L^{\pm}(m_W)\left(1 + \frac{\gamma^{\pm}}{2\beta_0}\frac{g_s(m_W)}{4\pi}\ln\frac{m_b^2}{m_W^2} + O\left(g_s^2\right)\right)$$

DOES WORK EVEN IF LOGARITHM IS LARGE
PERTURBATIVE CALCULATION NOT SPOILED!

$$\begin{pmatrix} L_1(m_b) \\ L_2(m_b) \end{pmatrix}_{LL} \simeq \begin{pmatrix} 1.02 & -0.11 \\ -0.48 & 1.18 \end{pmatrix}\begin{pmatrix} L_1(m_W) \\ L_2(m_W) \end{pmatrix}$$

SUMMARY OF OPERATOR RUNNING & MIXING

1) TREE-LEVEL MATCHING RELATION AT $\mu_1 \simeq m_W$
2) COMPUTE 1-LOOP CORRECTIONS IN LEFT
3) THESE LEAD TO RGEs THAT RESUM TO ALL ORDERS LARGE LOGS γ $\alpha_s \ln \left(m_W^2 / m_b^2 \right)$ $\mu_2 \simeq m_b$

4) AT FIXED-ORDER, RESULTS OF SM CALCULATION REPRODUCED
5) RGE EFFECTS MIX OPERATORS AMONG THEM

ASSUME

$$\begin{pmatrix} L_1(m_b) \\ L_2(m_b) \end{pmatrix} \simeq \begin{pmatrix} 1.02 & -0.11 \\ -0.48 & 1.18 \end{pmatrix} \begin{pmatrix} 0 \\ L_2(m_W) \end{pmatrix}$$

$$\boxed{L_1(m_b) = -0.11\, L_2(m_W)}$$

EFFECTS ARE $\mathcal{O}(\alpha_s)$

TAKE AWAY MESSAGE

↳ RELEVANCE IN EFT OF ACCOUNTING <u>ALL</u> OPERATORS, ALSO THOSE WHICH VANISH AT MATCHING SCALE

THESE CONSIDERATIONS ARE RELEVANT FOR ALL EFTs WE WILL SEE IN THIS COURSE