THE WEAK EFT 15/6/2023

THE STANDARD MODEL IS COMPOSED BY PARTICLES SPANNING A BROAD RANGE OF MASS

$m_e = 0.5\,MeV \longrightarrow m_t \sim 175\,GeV$

IN PARTICULAR, THE WEAK GAUGE BOSONS WHOSE MASSES ARISE FROM EWSB HAVE MASSES

$m_V \sim O(v)$, WITH v HIGGS VEV 246 GeV

$$v = \frac{2M_W}{g_{EW}}$$

g_{EW}: WEAK ISOSPIN COUPLING

$M_W \sim 80\,GeV \qquad M_Z \sim 91\,GeV$

FOR PROCESSES INVOLVING WEAK INTERACTIONS BUT WHERE THE TYPICAL MOMENTUM TRANSFER IS

$p \ll M_W$

THE WEAK BOSONS NEVER APPEAR AS EXPLICIT DEGREES OF FREEDOM IN SCATTERING AMPLITUDES AND CAN BE "INTEGRATED OUT"

SM $\xrightarrow{\text{INTEGRATE OUT } W, Z, t, h}$ WEAK EFT

u, d, s, c, b, t $\longrightarrow$ u, d, s, c, b

e, μ, τ $\longrightarrow$ e, μ, τ

W, Z, h

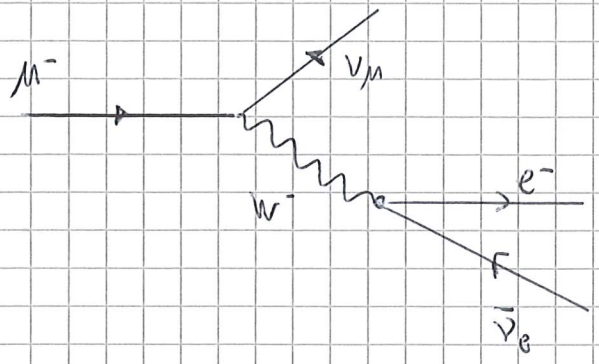
EFFECTS OF INTEGRATED OUT PARTICLES

↳ DETERMINE EFFECTIVE COUPLINGS & INTERACTIONS

PARADIGMATIC APPLICATION OF WEFT

↳ FERMI THEORY OF WEAK INTERACTION

CONSIDER WEAK DECAY OF MUON IN SM



Q WHAT ARE TYPICAL MOMENTUM TRANSFER INVOLVED IN MUON DECAY?

$$p_W \sim O(m_{\mu^-}) \ll M_W$$

→ INTERMEDIATE W BOSON CAN BE INTEGRATED OUT

W-BOSON → CHIRAL INTERACTIONS

[COUPLES ONLY TO LEFT-HANDED FERMIONS]

$$\mathcal{L}_{SM} \supset -\frac{g_2}{\sqrt{2}} W^+_\mu \, j^\mu_W + h.c.$$

$$j^\mu_W = \sum_{\ell=e,\mu,\tau} \bar{\nu}_\ell \gamma^\mu P_L \ell + \text{quark current}$$

P_L → PROJECTION TO LEFT-HANDED CHARGED LEPTONS

g_2 → $SU(2)_L$ COUPLING

$$P_{R(L)} = \frac{1 + (-)\gamma^5}{2}$$

Q DO I NEED TO APPLY A P_L PROJECTION OPERATOR TO NEUTRINO FIELD? WHY NOT?

LET US COMPUTE TREE LEVEL AMPLITUDE FOR MUON DECAY IN THE SM

$$i\mathcal{M}\left(\mu^-(p_1) \to \nu_\mu(p_2) + e^-(p_3) + \bar{\nu}_e(p_4)\right)$$

$$= \left(-i\frac{g^2}{\sqrt{2}}\right)^2 \left[\bar{u}_{\nu_\mu}(p_2)\,\gamma^\mu\, P_L\, u_\mu(p_1)\right] \times$$

↑ LEFT-HANDED SPINOR

$$\times \underbrace{\left(\frac{-i}{p^2 - M_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right)\right)}_{\text{W-PROPAGATOR IN UNITARY GAUGE}} \times \left[\bar{u}_e(p_3)\,\gamma^\nu\, P_L\, v_{\bar{\nu}_e}(p_4)\right]$$

$$p^\mu = p_1^\mu - p_2^\mu$$

IN THE FULL SM, WE WOULD NOW PROCEED WITH USUAL DIRAC ALGEBRA

Q HOW CAN I USE HERE THE FACT THAT $m_\mu \ll m_W$ IN ORDER TO SIMPLIFY CALCULATION?

→ $p^2/M_W^2 \ll 1$ $\quad (p \sim \mathcal{O}(m_\mu))$

↳ USE AS EXPANSION PARAMETER

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \left(\frac{p^2}{M_W^2}\right)^2 + \dots\right)$$

↑ NO DEPENDENCE ON W MOMENTUM AT ALL!! ← | LEADING TERM IN EFT EXPANSION

HENCE LEADING TERM IN EFT EXPANSION

$$D^{W}_{\mu\nu}(p) = \frac{i}{M_W^2} + O\left(p^2/M_W^2\right)$$

$$i\mathcal{M}\left(\mu^- \to \nu_\mu + e^- + \bar{\nu}_e\right) = -\frac{i g_2^2}{2M_W^2} \times$$

$$\times \bar{v}_{\nu_\mu}(p_2)\,\gamma^\mu P_L\, u_\mu(p_1)\; \bar{u}_e(p_3)\,\gamma_\mu P_L\, v_{\nu_e}(p_4)$$

WE SEE THAT THIS DECAY AMPLITUDE DOES NOT CONTAIN ANY DEPENDENCE WITH p^μ_W

↳ MUON DECAY DESCRIBED IN TERMS OF FOUR FERMION OPERATORS

$$\mathcal{L}_{WEFT} \supset \frac{C}{\Lambda^2}\left(\bar{\nu}_\mu \gamma^\mu P_L \mu\right)\left(\bar{e}\,\gamma_\mu P_L\, \nu_e\right) + h.c. + \dots$$

WILSON COEFFICIENT

LARGE SCALE, REGION OF EFT VALIDITY

SINCE WE KNOW THAT THE SM IS THE "FUNDAMENTAL THEORY" OF THE WEAK EFT, TREE LEVEL MATCHING TELLS US THAT

$$\frac{C}{\Lambda^2} = -\frac{g_2^2}{2M_W^2}$$

Q HOW WE COULD HAVE USED A BOTTOM-UP APPROACH, IF WE DID NOT KNOW THE EXISTENCE OF W BOSON, TO PUT A BOUND ON W MASS?

ASSUME MEASUREMENT OF MUON DECAY WIDTH

ASSUMING MUON DECAY VIA FOUR FERMION INTERACTIONS THE WEFT LAGRANGIAN IS TYPICALLY EXPRESSED IN TERMS OF FERMI CONSTANT

$$\mathcal{L}_{WEFT} \supset -\frac{4 G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e} \gamma_\mu P_L \nu_e)$$

$[G_F] = -2$

G_F CAN BE EXTRACTED FROM MUON DECAY WIDTH

$$\Gamma = \frac{G_F}{192 \pi^3} m_\mu^5$$

ON GENERAL GROUNDS

$$G_F \propto \frac{C}{\Lambda^2}$$

↓ SCALE WHERE WEFT BREAKS DOWN

$$C \leq 1 \rightarrow \Lambda^2 \leq \not{C} G_F^{-1}$$

SETS UPPER BOUND ON "SCALE OF NEW PHYSICS"

PUTTING BACK ALL NUMERICAL CONSTANTS

$$G_F = \frac{\sqrt{2}\, g^2}{8 M_W^2} = \frac{1}{\sqrt{2} v^2}$$

$$\Lambda^2 \leq v^2 \sqrt{2} \qquad \boxed{\Lambda \leq 287.8 \text{ GeV}}$$

MUON DECAY MEDIATED BY FOUR-FERMION OPERATORS PREDICTS "NEW PHYSICS" BEFOR 300 GeV!

THE EXPANSION PARAMETER OF WEAK EFT IS

$$\delta \sim p/M_W$$

$O(\delta^0)$ ⟶ DIMENSION-6, FOUR FERMION OPERATORS

$O(\delta^2)$ ⟶ DIMENSION-8, " " " WITH DERIVATIVE COUPLINGS

GENERAL → WEAK EFT ALSO KNOWN AS

LEFT / LOW-ENERGY EFT)

↳ ALL PARTICLES HEAVIER THAN m_b → INTEGRATED OUT

(no t, h, W, Z)

Q → WHAT SYMMETRIES MUST THIS LEFT/ WEFT OBEY?

PHYSICS BELOW EW SYMMETRY BREAKING SCALE

SM ABOVE v

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

SM BELOW v & LEFT

$$SU(3)_C \otimes U(1)_{EM}$$

ACCIDENTAL SYMMETRIES OF SM

- BARYON NUMBER CONS
- LEPTON-FAMILY-NUMBER CONSERVATION

↳ INHERITED BY LEFT WHEN MATCHED TO SM

Q BOTTOM-UP CONSTRUCTION OF LEFT SHOULD SATISFY THESE ARTIFICIAL SYMMETRIES? → NO!!